MATH 590: QUIZ 2

Name:

Throughout V will denote a vector space over F, where $F = \mathbb{R}$ or $F = \mathbb{C}$.

1. For subspaces $W_1, W_2 \subseteq V$, define what it means for V to be the *direct sum* of W_1 and W_2 and give an example of a direct sum decomposition. (4 points)

Solution. V is the direct sum of the subspaces W_1 and W_2 if the following conditions hold: (i) $V = W_1 + W_2$ and (ii) $W_1 \cap W_2 = \vec{0}$.

Possible examples: $\mathbb{R}^2$ is the direct sum any any two distinct lines though the origin, in particular, $\mathbb{R}^2$ is the direct sum of the x-axis and y-axis. Similarly, $\mathbb{R}^3$ is the direct sum of any plane through the origin and line through the origin not lying in that plane. In particular, $\mathbb{R}^3$ is the direct sum of the xy-plane and the z-axis.

2. For $v_1, \ldots, v_r \in V$ state two equivalent conditions for this set of vectors to be linearly dependent. (4 points)

Solution. The vectors $v_1, \ldots, v_r$ are linearly dependent if the following equivalent conditions hold:

(i) There exist $a_1, \ldots, a_r \in F$, not all 0, such that $a_1v_1 + \cdots + a_rv_r = \vec{0}$.

(ii) For some $1 \leq i \leq r$, $v_i \in \text{Span}\{v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_r\}$.

3. Describe the subspace of $M_{2\times 2}(\mathbb{R})$ spanned by the matrices $\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}$. (2 points)

Solution. The span of the given matrices is the set of all matrices of the form $\begin{pmatrix} 2a & -a \\ -b & b \end{pmatrix}$ with $a, b \in \mathbb{R}$.